

Recent developments on the application of discrete symmetries

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J E Kim. “Discrete symm:Mother of global symm”@HighOne, 26 Jan 2015. 1/67



Influentials lately



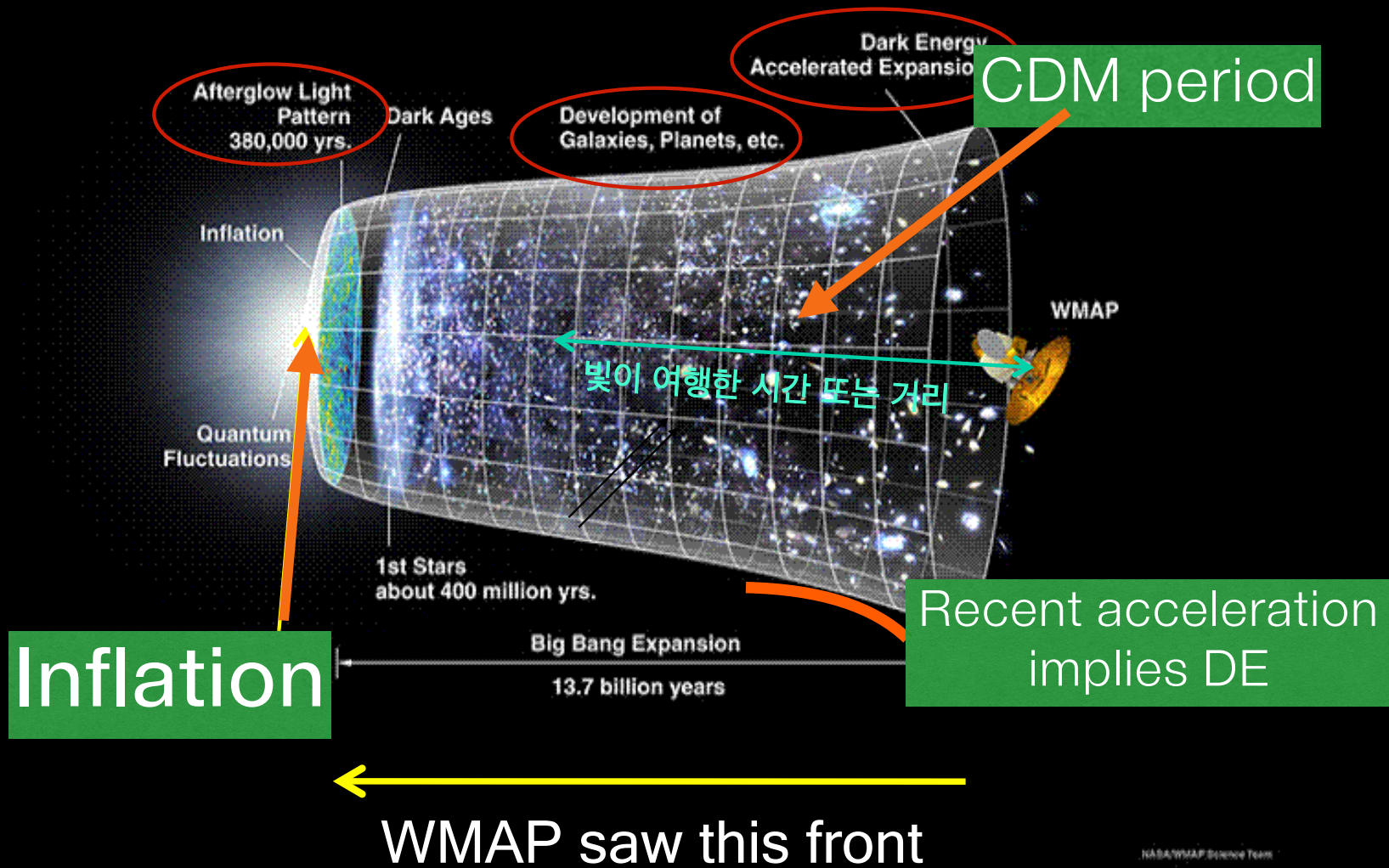
D K Hong Aug 2008



← 평화의 전당 ICHEP 2018

S Nam, Apr. 2013





E right after sunset

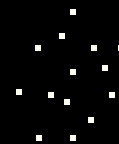
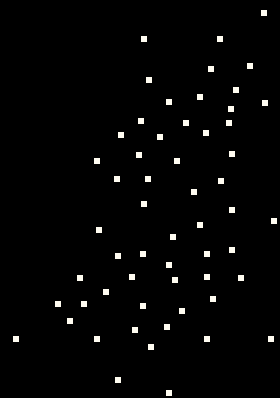


W around 7:30pm at
Sutherland dorm area

Southern sky to the 60 degree
on the way back to the hotel
from the observatory

W around 8:30 outside
the visitor center





Southern sky to the 60 degree
on the way back to the hotel
from the observatory. Hydra



Southern sky to the 60 degree
on the way back to the hotel
from the observatory. Hydra

1. ~~With discrete symmetry~~ Discrete symmetry is the key.
2. ~~With QCD axion~~ Includes QCD axion.
3. The strong CP prob. is solved and the very light axion provides some CDM component and even a lighter one for DE.

Applied to high scale inflation

4. High scale inflation needs $NDW=1$
5. Possible in string comp. with anomalous $U(1)$
6. CAPP may get an answer to axion in the micro- to milli-eV range, even with 10% of CDM



- Introduction
- Axions and the strong CP problem
- Discrete symmetry on the bottom of global symmetries
- Dark energy from $U(1)_{DE}$
- Gravity waves with large r
- PQ symmetry breaking below H_I



1. Introduction



DE as BCM review
Kim-Semertzidis-Tsujikawa,
1409.2497 [hep-ph]

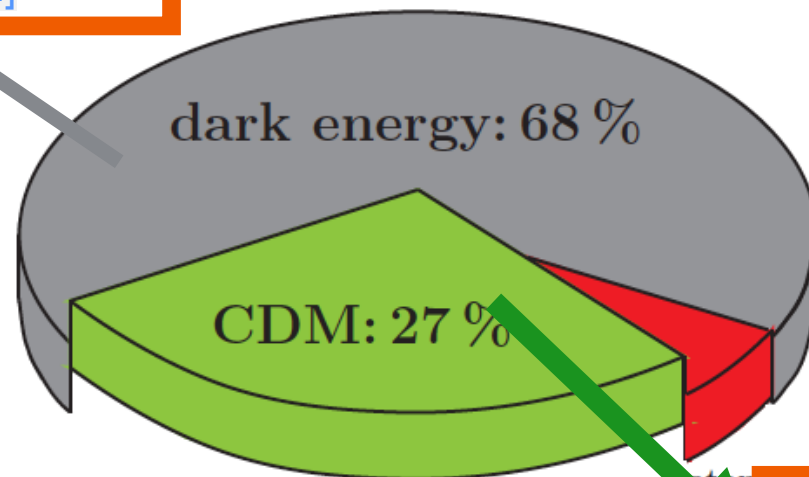


Figure 1.1: Cosmic energy pie

Yes

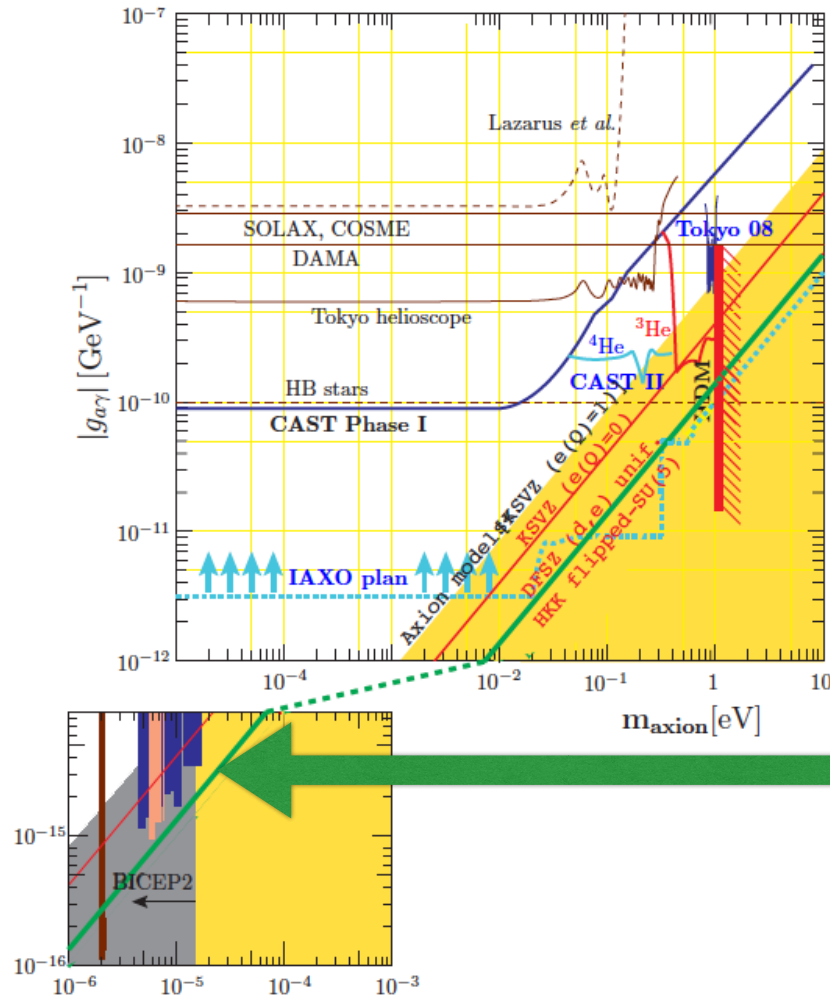
2 ethers?

No

Non-thermal DM review,
Baer et al. Phys. Rep. 555 (2015) 1
doi:10.1016/j.physrep.2014.10.002

BCM review
Kim-Semertzidis-Tsujikawa,
Front. Phys. 2 (2014) 60
1409.2497





$g_{a\gamma}(= 1.57 \times 10^{-10} c_{a\gamma\gamma})$ vs. m_a plot

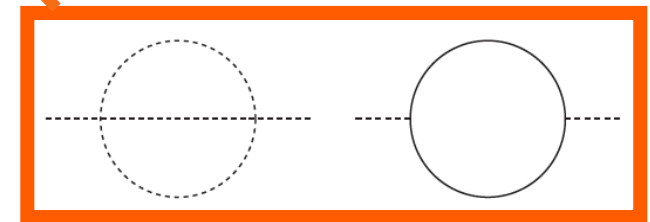
Kim-Semertzidis-Tsujikawa,
Front. Phys. 2 (2014) 60

Kim, PLB 735 (2014) 95,
Corrigenda
[in HKK model with
SU(5)xSU(5)'
in Z(12-I) orb. comp. (2009)]
 $c(a\text{-photon-photon})=8/3$.



Ideas	Description [scalar S or pseudoscalar P]	Discr. sym.?	Fine-tuning?	Model from string
MOND ^a	Change Newtonian gravity. [No boson]	Irrelevant	Yes	Not yet
Anthropic principle	Out of many possible vacua, only those suitable for age $> t_U$ survived. [S or P]	Irrelevant	Irrelevant	Not yet ^b
Quintessence	With a runaway $V \propto 1/\phi^n$ ($n > 0$). [S]	No	Yes ^c	Not yet
Dilaton	P-Gold. boson from dilaton sym. [S]	No	Yes ^d	Not yet
$U(1)_{DE}$ Gold. boson	P-Gold. boson from $U(1)_{DE}$ sym. [P]	Yes	No	Yes ^e

If each vertex satisfies $U(1)$, the whole diagram also!!!



Kim-Semertzidis-Tsujikawa (2014):
Bosonic coherent motion review



Kim-Semertzidis-Tsujikawa, Front. Phys. 2, 60 (2014):
Bosonic coherent motion review
[Review in research topic BCM,
New one: Frontiers in Physics,
HE & astroparticle physics]

BCM:

BCM1 → Axion
BCM2 → Inflaton

CCtmp:
 ↓
Dark energy



2. Axions and the strong CP problem



- Because of instanton solutions of QCD, there exists an effective interaction term $G \tilde{G}$ -dual. It is the flavor singlet and the source of solving the U(1) problem of QCD: 't Hooft, Phys. Rep. (1986).
- This term is physical, but leads to
- The strong CP problem, “Why is the $n\text{EDM}$ so small?”
- The remaining ‘natural solution’ is “very light axion”.



- The gluon interaction.

$$\mathcal{L} = \bar{\theta}\{G\tilde{G}\} \equiv \frac{\bar{\theta}}{64\pi^2} \epsilon^{\mu\nu\rho\sigma} G_{\mu\nu}^a G_{\rho\sigma}^a,$$

- The neutron mass term.

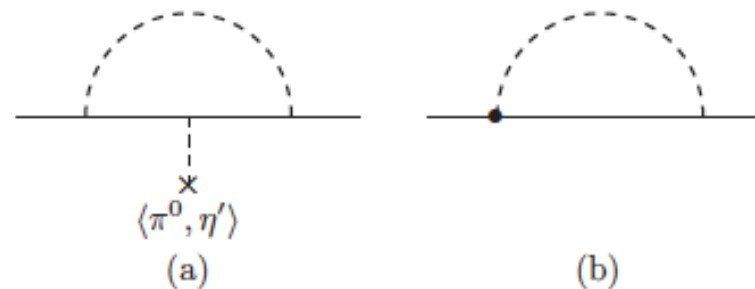
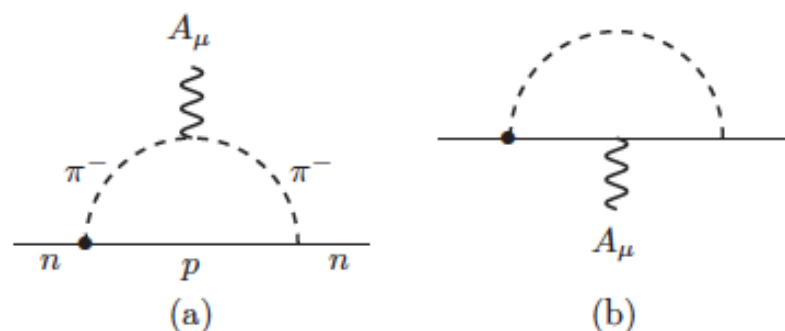


FIG. 4. Loop corrections for $\bar{n}n$ -meson coupling. Insertion of the CP violation effect by VEVs of π^0 and η' in (a). They can be transferred to one vertex shown as a bullet in (b). With this bullet, CP violation is present because of a mismatch between the CP -conserving RHS vertex and CP -violating LHS vertex.



- The neutron EDM term.



$$\frac{d_n}{e} = \frac{g_{\pi NN} \overline{g_{\pi NN}}}{4\pi^2 m_N} \ln\left(\frac{m_N}{m_\pi}\right),$$

FIG. 5. Diagrams contributing to the NEDM with the bullet representing the CP violation effect. (a) is the physically observable contribution.

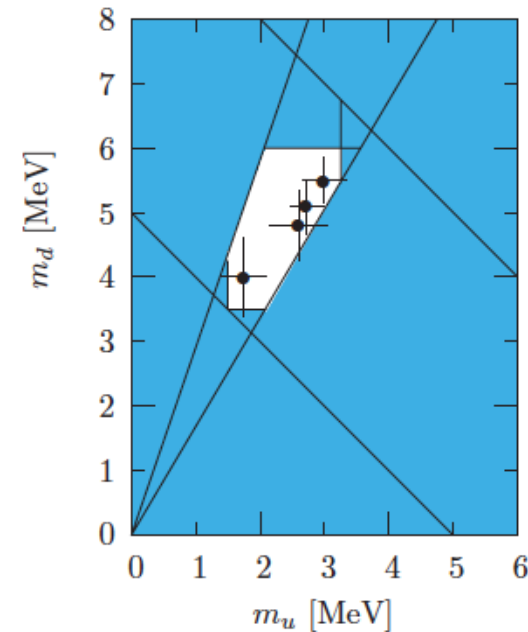
Pion VEV (CP violating)
in the meson L can give

$$\overline{g_{\pi NN}} = -\bar{\theta} \frac{Z}{(1+Z)} \simeq -\frac{\bar{\theta}}{3}.$$



Masses up quark:

PDG book,
Manohar-Sachrajda

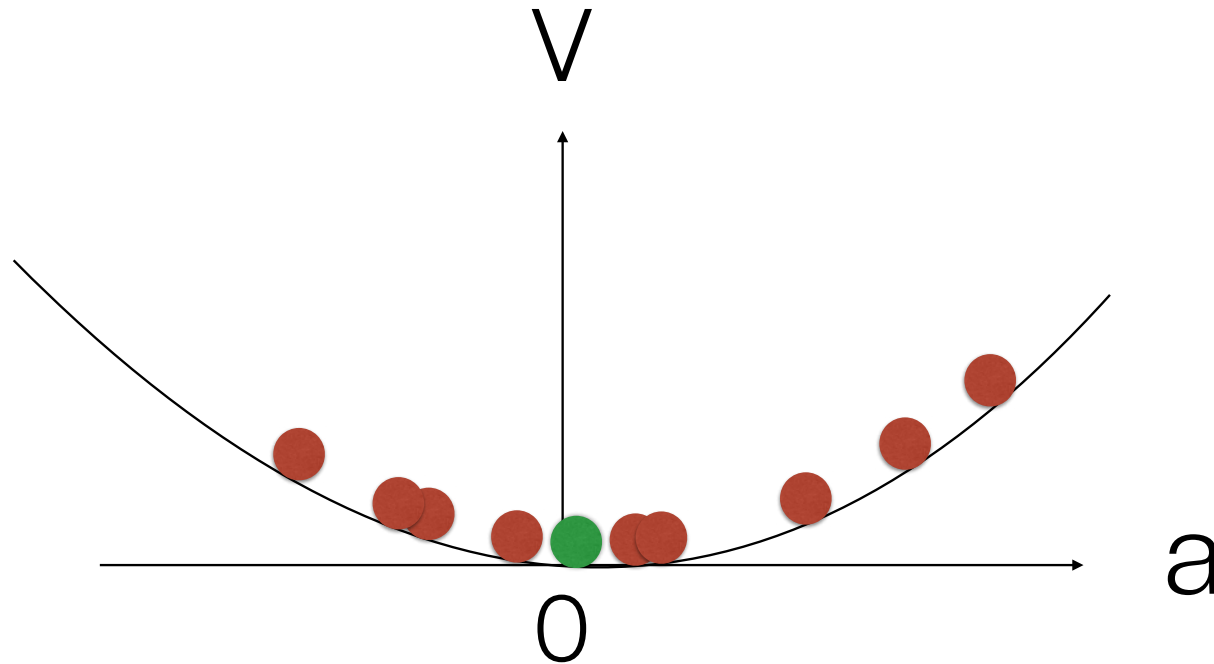


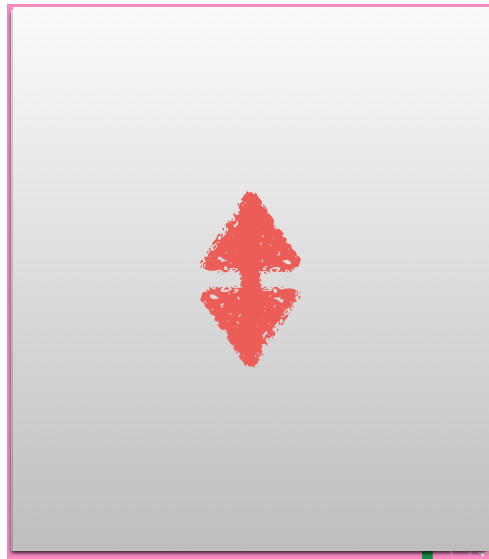
Symmetry solution (natural solution, calculable solution):

Beg-Tsao, Georgi, MS, ..., Nelson, Barr:
But no compelling model with very small θ -bar



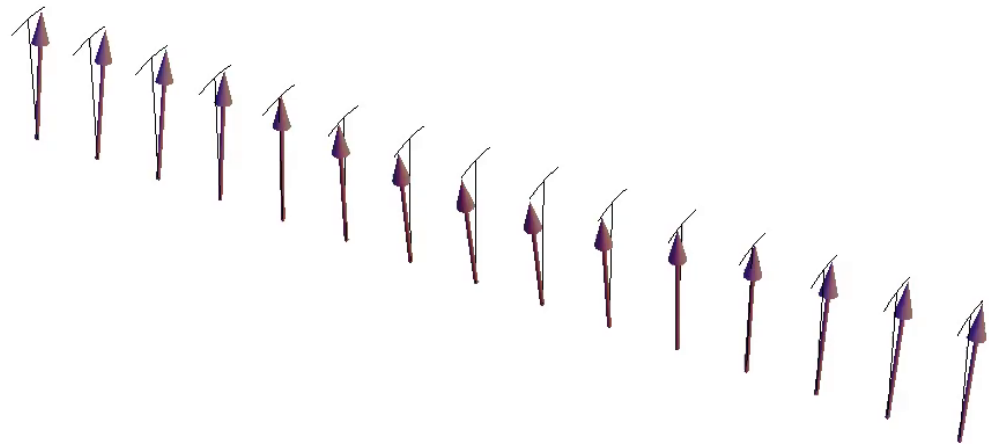
Axion solution = cosmological solution



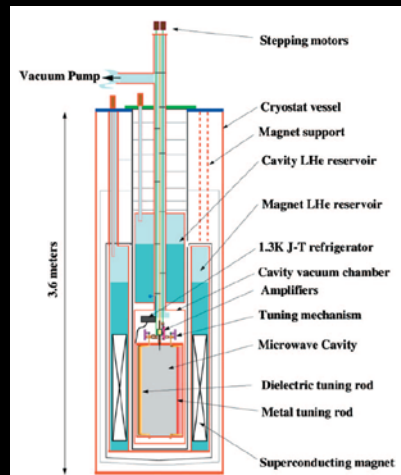


Axion detection scheme

Constant B field
E field follows the
axion oscillation



Very light axion : 1979
CDM candidate



ADMX

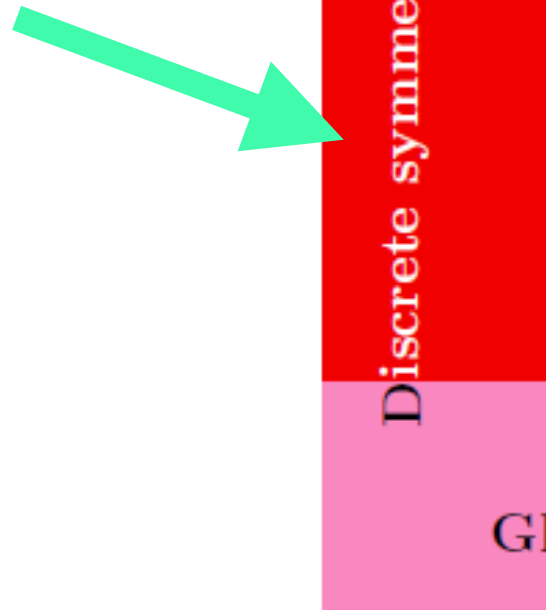
CAPP: ???

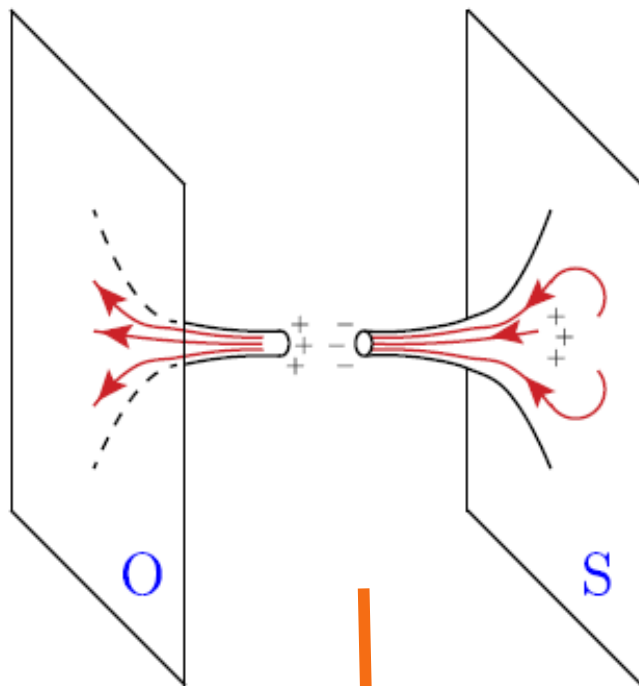
PQ symmetry : 1977
Detection suggested : 1983
CAPP started : 2013

3. Discrete symmetries on the bottom of global symmetries



The dominant contribution is
QCD anomaly term





Wormholes:
Gidding-Strominger,
Coleman, Cline

For PQ,
Barr-Seckel,
Kamionkowski-MarchRussel,
Holdom et al.
Exclude terms up to dim 8.

The example of acc symm.

Discrete gauge symmetry:
Ibanez-Ross,
Krauss-Wilczek



★ We can think of two possibilities of discrete symmetries realized from string compactification, below M_P :

(i) The discrete symmetry arises as a part of a gauge symmetry.

[Krauss-Wilczek, PRL 62 (1989) 1211]

We choose this.

(ii) The string selection rules directly give the discrete symmetry.

[JEK, PRL 111 (2013) 031801]



- All global symmetries are NOT exact!!!
- Question is “What is the explicit breaking terms?”
- If it is spontaneously broken by VEVs of scalar fields, then there is “decay constant f .”
- So we have two scales: Λ and f .
- Anomaly breaking is, probably numerically, the dominant piece.



Axion-photon-photon coupling

1. KSVZ and DFSZ are not the cases from string compactification.
2. Several couplings are  from Kim (1998)

TABLE I. $c_{a\gamma\gamma}$ in several field theoretic models. The left block is for the KSVZ and the right block is for the DFSZ. (m,n) in the KSVZ block denotes m copies of $Q_{\text{em}}=\frac{2}{3}$ and n copies of $Q_{\text{em}}=-\frac{1}{3}$ heavy quarks with the same PQ charge. In the DHSZ block $x=\tan\beta=v_u/v_d$.

Q_{em}	$c_{a\gamma\gamma}$	x	one Higgs couples to	$c_{a\gamma\gamma}$
0	-1.95	any x	(d^c, e)	0.72
$\pm\frac{1}{3}$	-1.28	any x	(u^c, e)	-1.28
$\pm\frac{2}{3}$	0.72			
± 1	4.05			
(m, m)	-0.28			



Approximate PQ symmetry:

K S Choi-I W Kim-JEK, JHEP 0703 (2007) 116, Z(12-I) orb. comp.;
Choi-Nilles-RamoSanches-Vaudrevange , PLB675 (2009) 381, Z(6-II)

When we use the anomalous U(1), the PQ symmetry is not the accidental one.

Strategy: Findout compactifications with anomalous U(1).

If it is a GUT, it is calculable since there are relatively a small number of representations.

It is done only with a flipped SU(5)
[JEK, PLB735, 95. arXiv:1405.6175].

It gives an exact U(1)PQ, but spontaneously broken.

SU(5)-flip xSU(5)'xSU(2)' x six U(1)s

$$Q_{\text{anom}} = 84Q_1 + 147Q_2 - 42Q_3 - 63Q_5 - 9Q_6.$$

$$\begin{aligned} Q_1 &= (0^5; 12 \ 0 \ 0)(0^8)', & \tilde{Q}_1 &= \frac{1}{12} Q_1, \\ Q_2 &= (0^5; 0 \ 12 \ 0)(0^8)', & \tilde{Q}_2 &= \frac{1}{12} Q_2, \\ Q_3 &= (0^5; 0 \ 0 \ 12)(0^8)', & \tilde{Q}_3 &= \frac{1}{12} Q_3, \\ Q_4 &= (0^8)(0^4, 0; 12 \ -12 \ 0)', & \tilde{Q}_4 &= \frac{1}{12\sqrt{2}} Q_4, \\ Q_5 &= (0^8)(0^5; -6 \ -6 \ 12)', & \tilde{Q}_5 &= \frac{1}{6\sqrt{6}} Q_5, \\ Q_6 &= (0^8)(-6 \ -6 \ -6 \ -6 \ 18; 0 \ 0 \ 6)', \end{aligned}$$



$$\sum_i Q(q_i)n(q_i) = -16 \quad -28 \quad +8 \quad 0 \quad +18 \quad +6 \quad -6984 = -17058$$
[illegible]

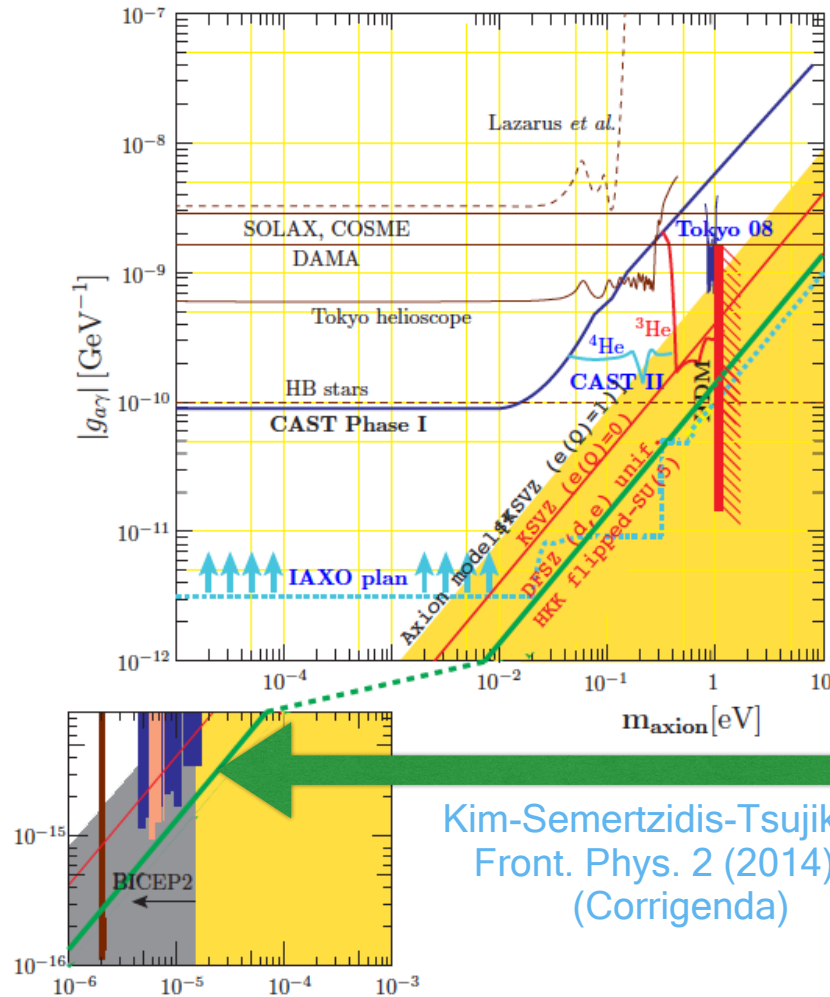
$\sum_i Q(Z_i) n(Z_i)$	-16	-28	$+8$	0	$+18$	$+6$	-6984	-736	m	Label	-882
10_0	1		-2	-2	-2	0		-648	$3T'_1$	0	-756

$(5', 2')_0$	1	-2	-2	-2	0	+3	-3	-540	$2F'_1$	0	+1008 +1008
											0
											0
$\bar{5}'_0$	1	-2	-2	-2	0	+3	-15	-432	F'_2	0	-1188 +1188

all:

$\bar{5}'_{-5/3}$	1	+4	-4	0	+4	+1	+11	-414	F'_3	-230	
	$\sum_i Q(1_i)n(1_i) =$	-16	-28	+8	0	+18	+4	-7632	$\sum_i =$	-460	-1296
$5'_{-5/3}$	3	-2	+2	+6	+4	-2	+2	-16	$3F_4$	-10	-1296
											-17058
$\bar{5}'_{5/3}$	3	-2	-6	+2	-4	+2	-2	-1242	$3F_5$	-690	
$\frac{1}{2}'$											
$5'_{-5/3}$	1	+4	0	-4	-4	-1	-11	+666	F'_6	+370	
	$\sum_i Q(q'_i)n(q'_i) =$	-16	-28	+8	0	+18	+6	-6984	$\sum_i =$	-1960	





$g_{a\gamma}(= 1.57 \times 10^{-10} c_{a\gamma\gamma})$ vs. m_a plot

$c(\text{axion-photon-photon})$
 $= 8/3 - 1.98$

Kim, PLB 735 (2014) 95,
 Corrigenda
 [in HKK model with
 $SU(5) \times SU(5)'$

in Z(12-I) orb. comp. (2009)]
 $c(a\text{-photon-photon}) = 8/3$.

Kim-Semertzidis-Tsujikawa,
 Front. Phys. 2 (2014) 60
 (Corrigenda)



4. Dark energy from $U(1)_{DE}$



For DE, the potential is extremely flat; still the DE vacuum looks like contributing to cosmological constant.

Anomaly-free is a small part.

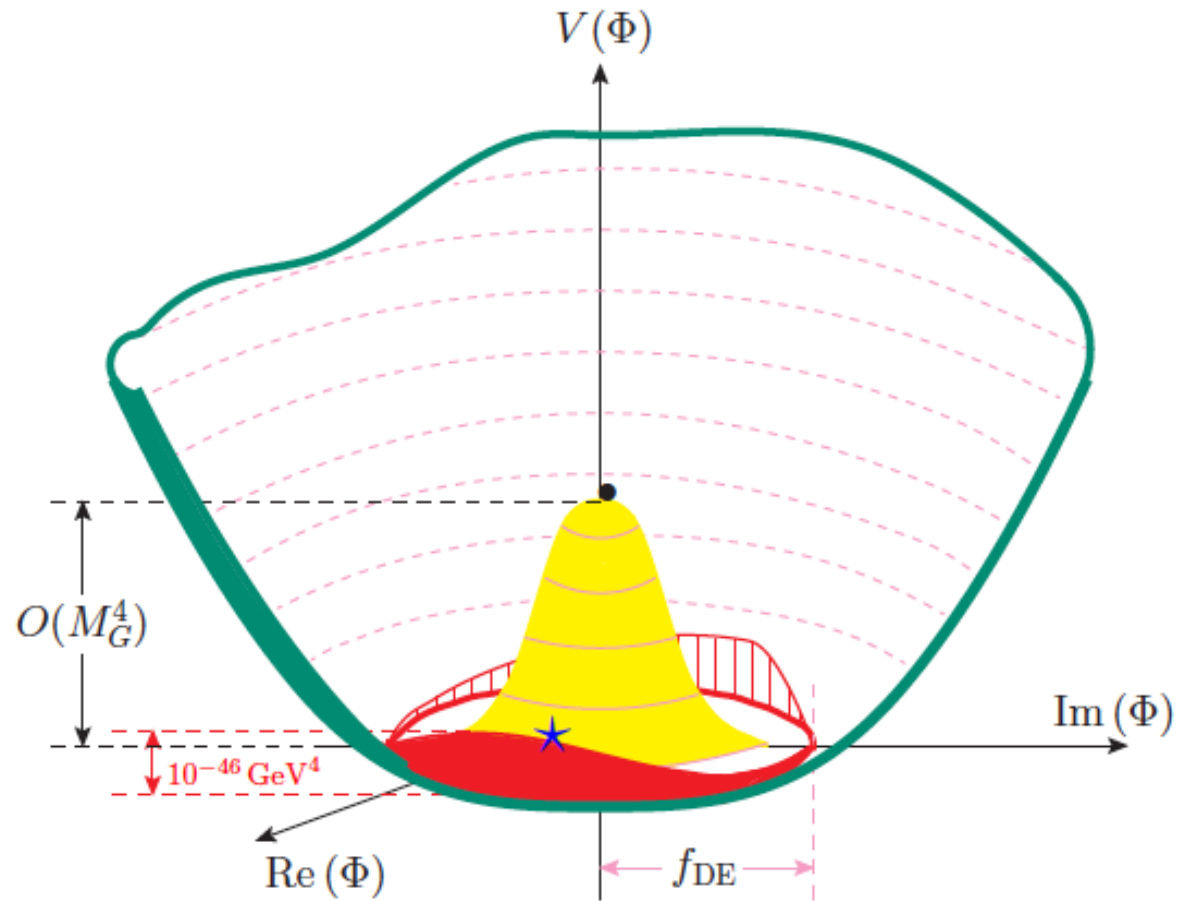
axion, the vacuum already started bosonic
notion(BCM) at $T_1 \approx 1$ GeV.

Anomaly is a large part.

How can one very flat and the other
By the red part!!!

To solve DE, we
necessarily introduce
QCD axion for it to
carry QCD anomaly.



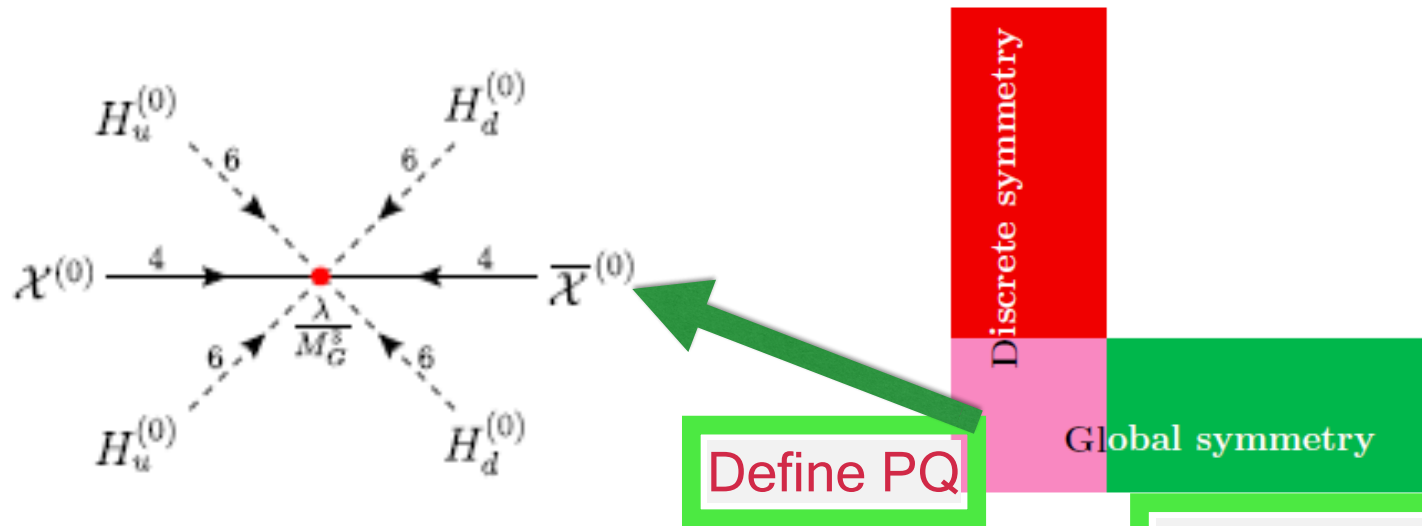


- $Z(12-I)$ is the simplest orbifold on 2-torus since there are only 3 fixed points. [Huh-K-Kyae, PRD 80, 115012 (2009)] Higgs with two units of discrete charge. 2 families from T_4 and 1 family from U . The gauge group is $\text{flipped-SU}(5) \times \text{SU}(5)'$ with extra (mostly) $U(1)$ groups.
- EM neutral singlets contain for example,

Sectors	Singlet states	χ	$(N^L)_j$	$\mathcal{P}(f_0)$	Label	P	Z_4	Z_8	Z_{10}	Z_{12}
T_3	$(0\ 0\ 0\ 0\ 0; \frac{-1}{2}\ \frac{-1}{2}\ \frac{-1}{2})(0\ 0\ 0\ 0\ 0\ \frac{3}{4}\ \frac{-1}{4}\ \frac{-1}{2})'$	L	0	1	S_9	-3	-4	-8	-10	-12
T_9	$(0\ 0\ 0\ 0\ 0; \frac{1}{2}\ \frac{1}{2}\ \frac{1}{2})(0\ 0\ 0\ 0\ 0\ \frac{-3}{4}\ \frac{1}{4}\ \frac{1}{2})'$	L	0	1	S_{13}	+3	+4	+8	+10	+12

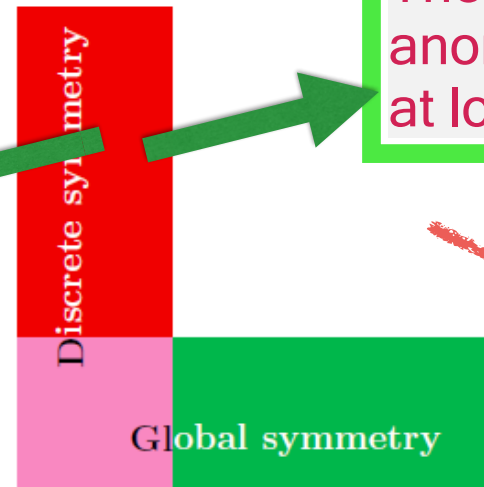
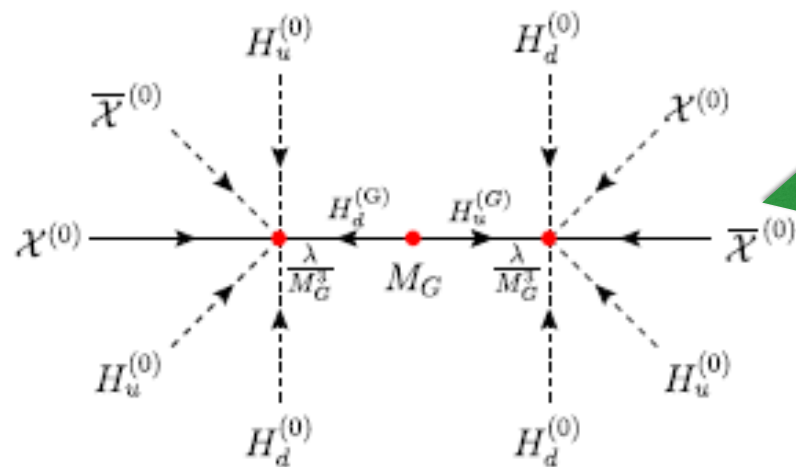
- $Z(10)R$ is quite large to forbid many terms in W .





- Higgs portal to high energy physics.
- This discrete symmetry charges are from the mother gauge U(1) charges.





The dominant anomaly piece at low energy: QCD

★ The height of the potential is highly suppressed and we can obtain 10^{-47} GeV^4 from discrete symmetry Z_{10R} , without the gravity spoil of the global symmetry breaking term.

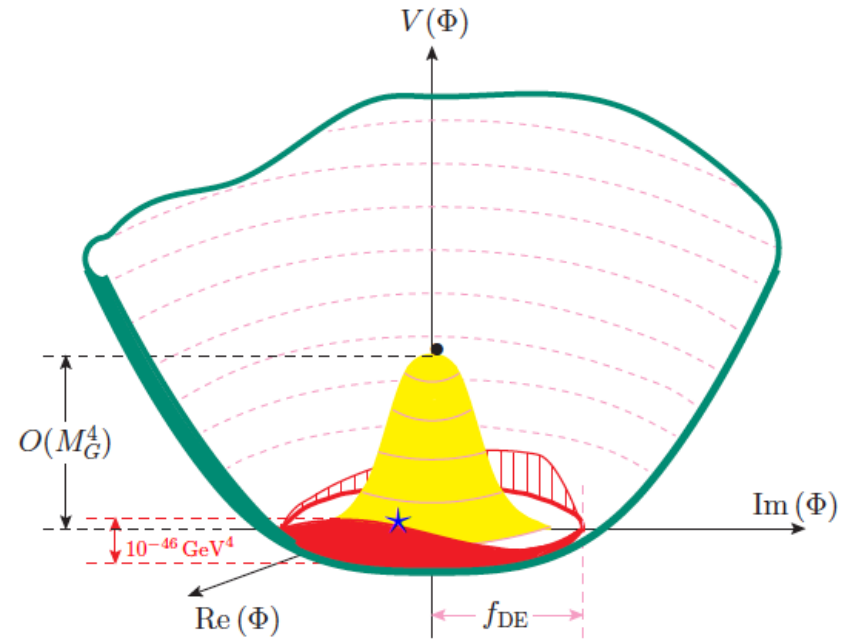


5. Inflation with trans-Planckian inflaton



The DE potential had the form of the potential with a GUT scale vacuum energy.

It predicts a hilltop inflation.



But there was the r and n_s values given by BICEP2. Is the hilltop consistent?

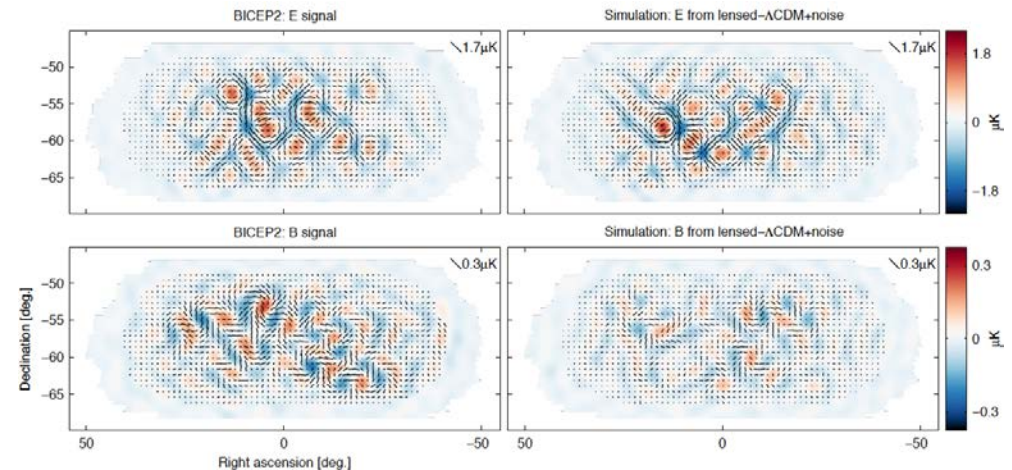


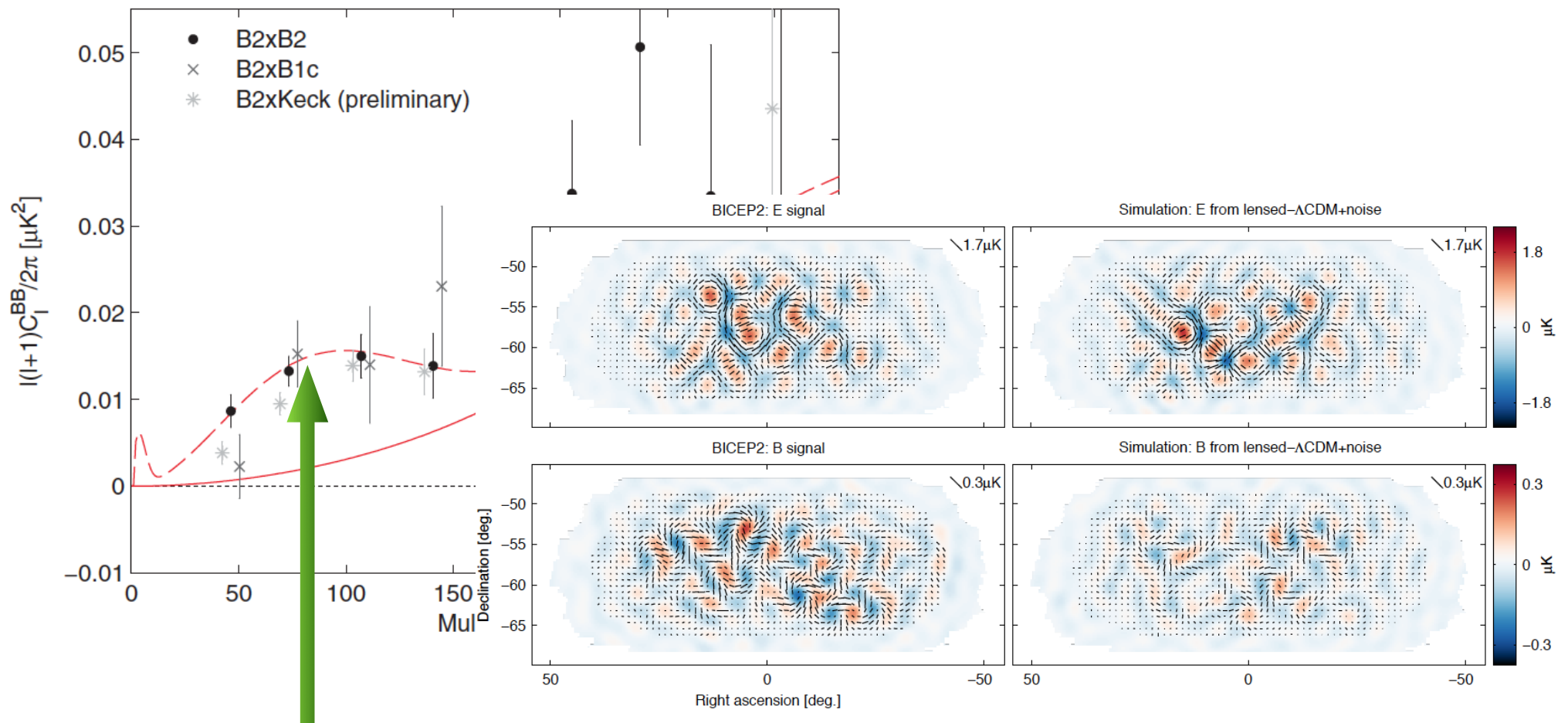
$$\nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t} \quad (\text{Faraday})$$

$$\nabla \times \mathbf{B} = \frac{4\pi}{c} \mathbf{j} + \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} \quad (\text{Ampere-Maxwell})$$

For large scales,
Gravity can give a
curl type as $\mathbf{B}$ has
a curl with current.

r = tensor:scalar ratio





For small l , r is measured.



★ We obtain

$$n_s = 1 + \Delta n_s = 1 - \left[\frac{r}{8} + \frac{1}{20s} + 2s \right] \leq 1 - \left[\frac{r}{8} + \sqrt{\frac{r}{2}} \right]$$

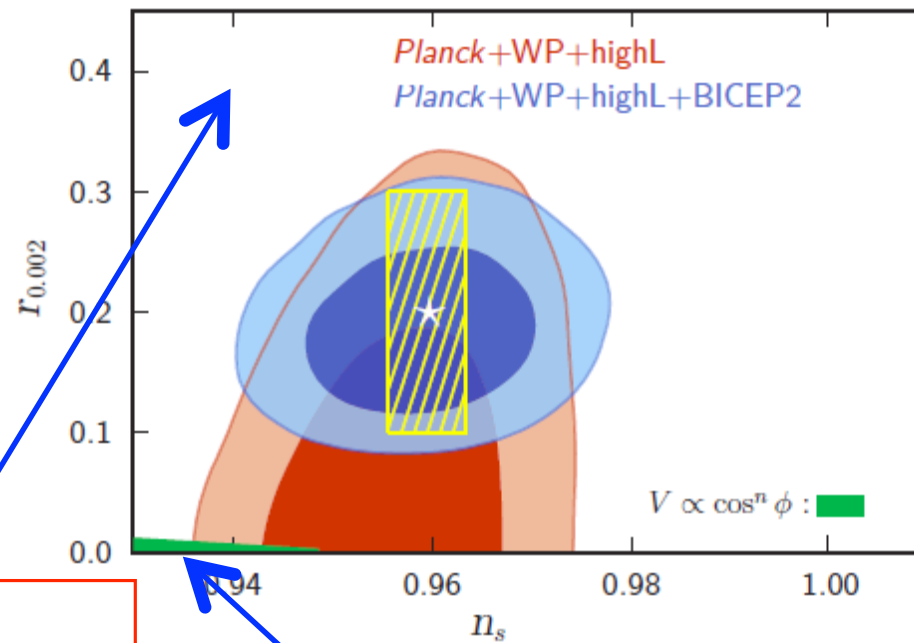
→ [0.96, 0.008]♪

$$V = \Lambda^4 \cos^s \phi + \Delta V$$

New type (chaoton)
hybrid inflation

with $(\Delta V)'' > 0$

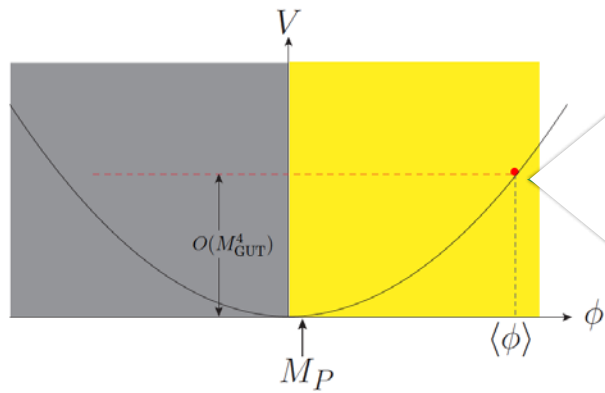




Natural inflation starting at π is here.
Freese-Kinney: 1403.5277.

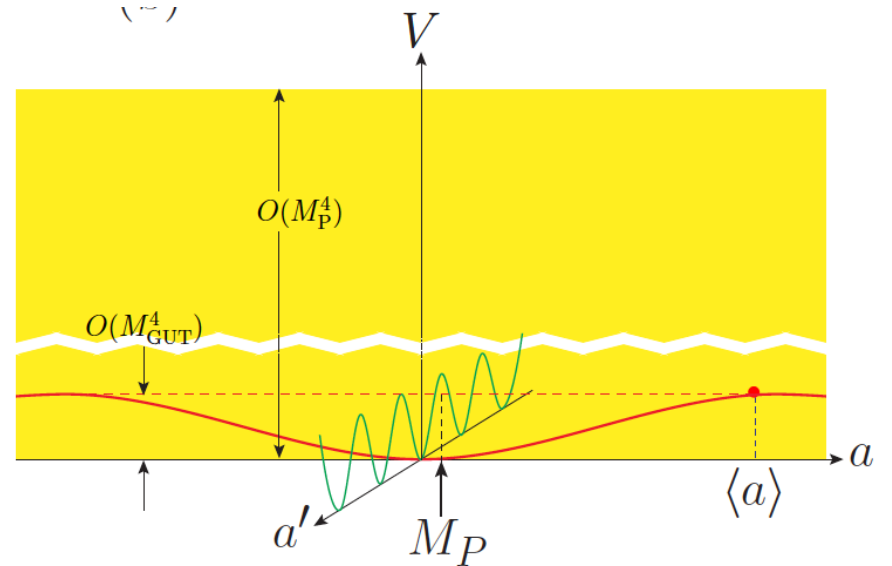
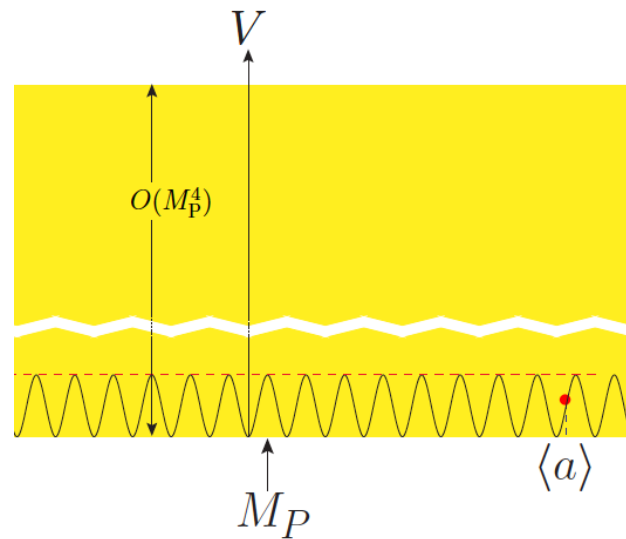
Natural inflation starting at 0 is here.





What happens here for higher order terms?

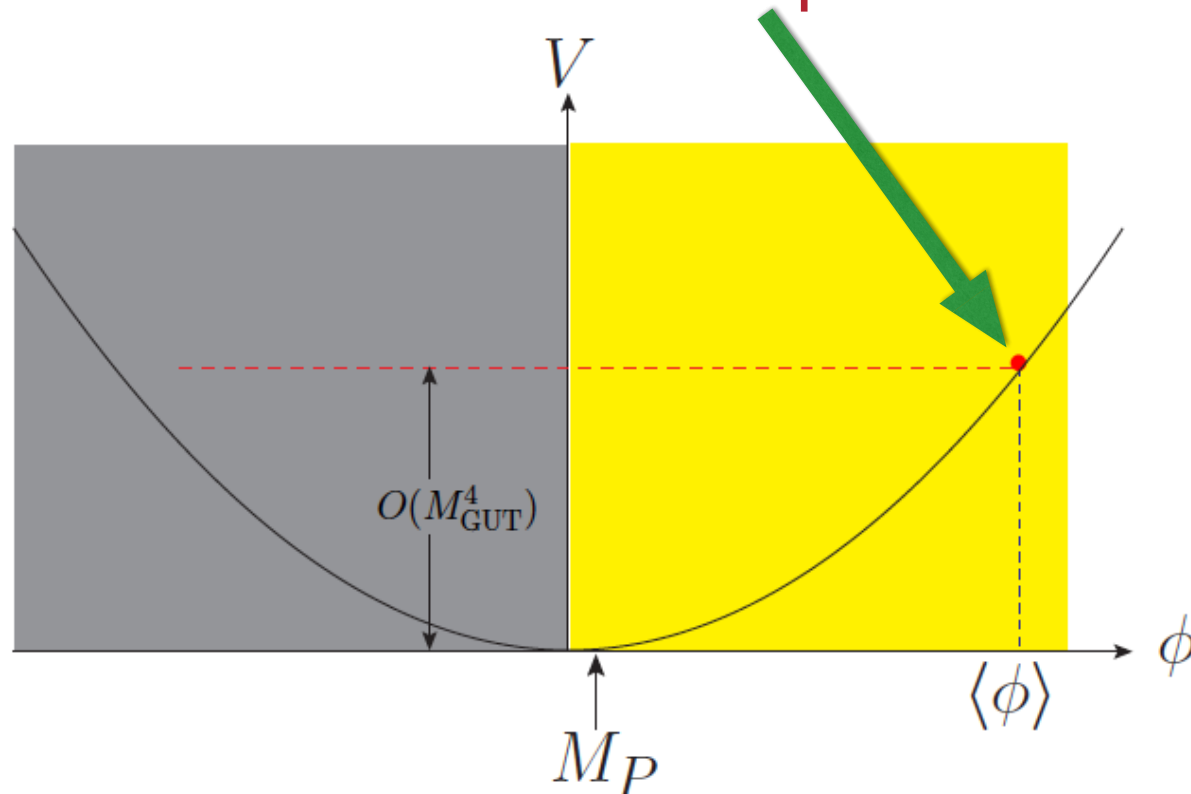
chaotic inflation with $m^2\phi^2$ [140],



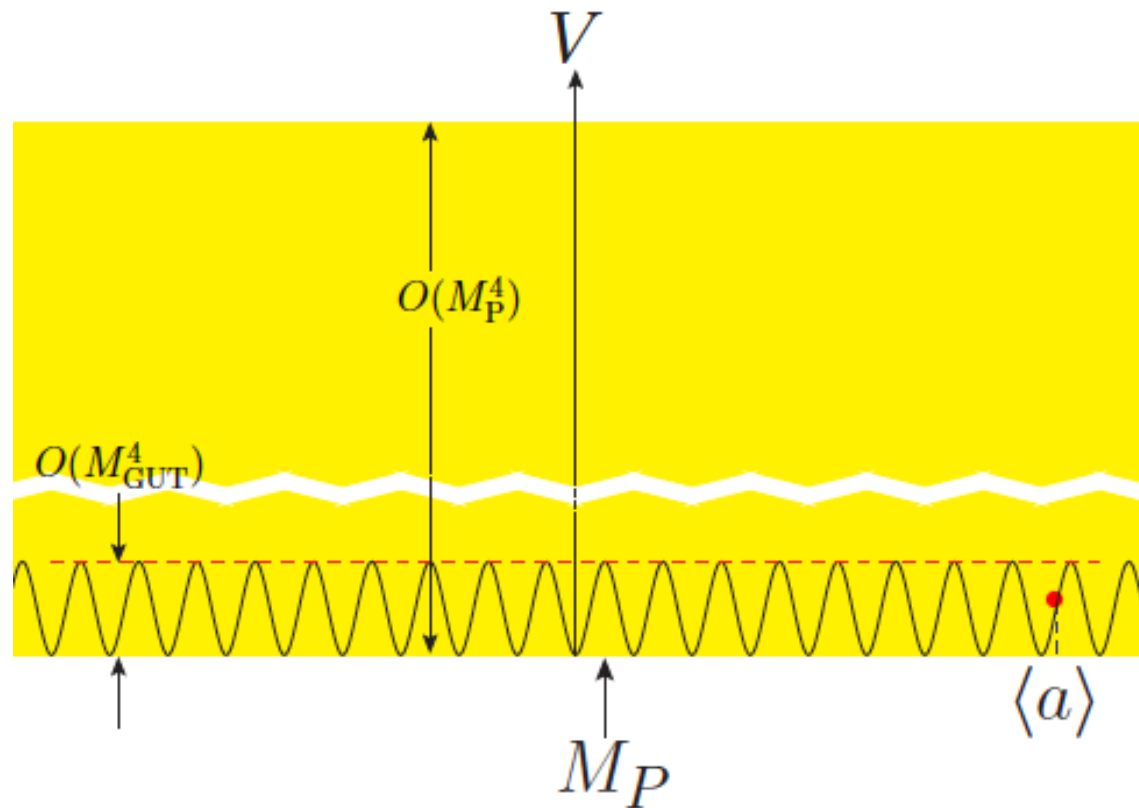
The natural inflation of [148]; (b) The KNP inflation [149].



Linde's chaotic inflation
needs a large field value
Lyth bound: $> 15 M_P$



Natural inflation suggested: [Freese-Frieman-Orlino]



$$V = \Lambda_{GUT}^4 \left(1 - \cos \frac{a_N}{f_N} \right).$$

Freese-Frieman-Orlinto (1990)

Decay constant f can be determined
by radial component

$$V = \frac{1}{4} \lambda (|\phi|^2 - f^2)^2.$$



10^{-6} for trans-Planckian

Then, it is the hill top inflation.
So, the radial field must roll down
the hill quickly: not inflaton



If we stick to a single field inflation,
the hilltop inflation is almost ruled out.

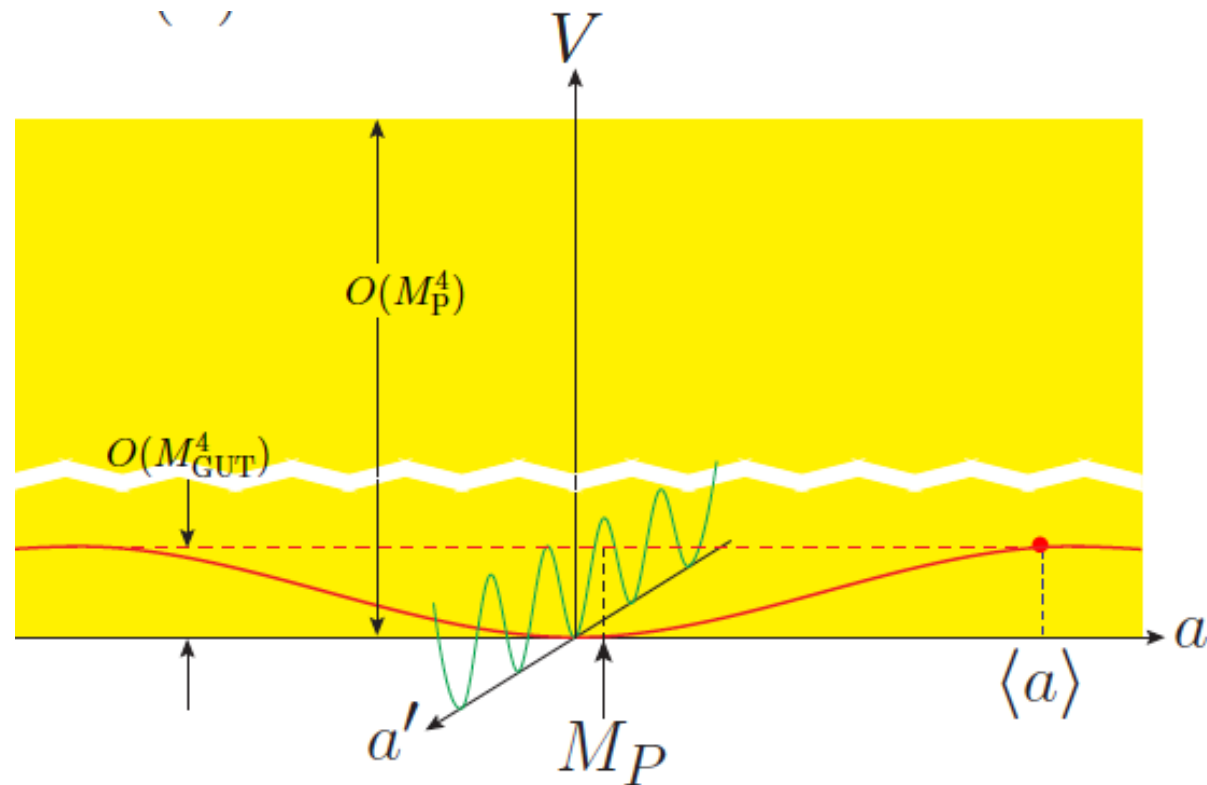
$$V(\phi) = \ell^4 \times \begin{cases} (\phi^2 - f_{\text{DE}}^2) - m^4 \\ \cos^s \phi \quad s = \text{even} \end{cases}$$

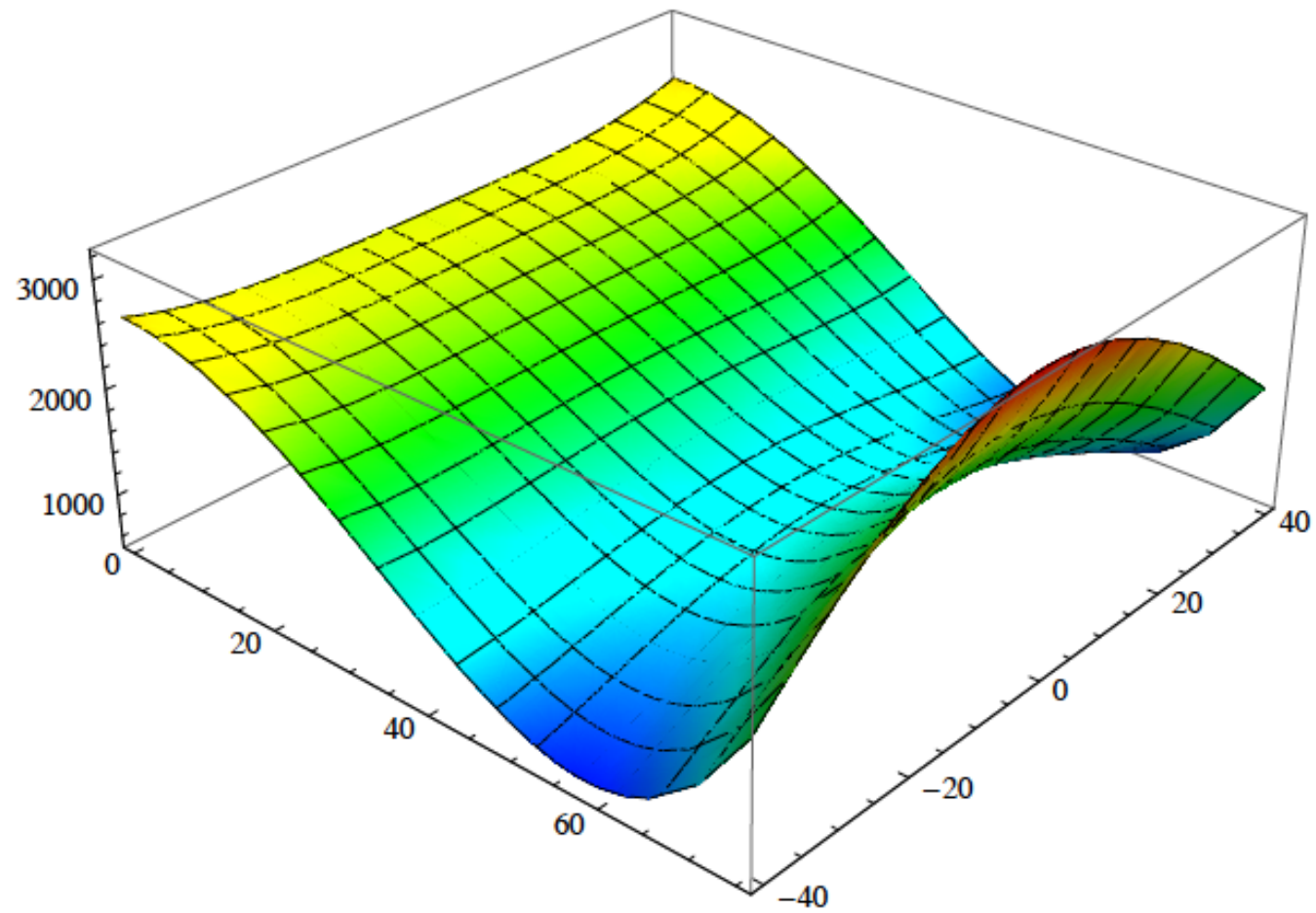
$$n_s = 1 + \Delta n_s = 1 - \left[\frac{r}{8} + \frac{1}{20s} + 2s \right] \leq 1 - \left[\frac{r}{8} + \sqrt{\frac{r}{2}} \right]$$

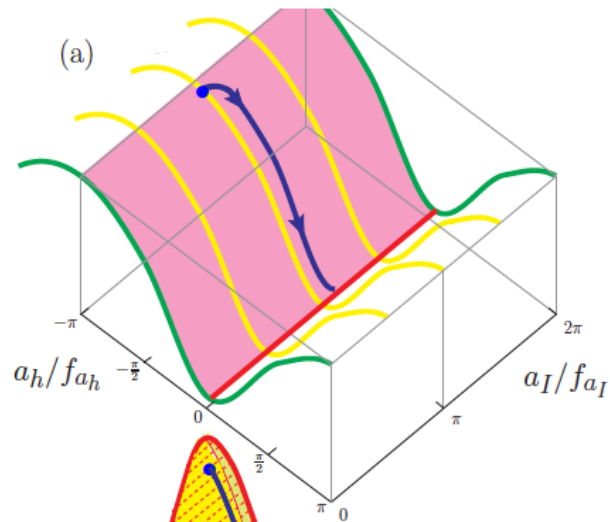
→ [0.96, 0.008] ♪



2-flation suggested: [Kim-Nilles-Peloso]

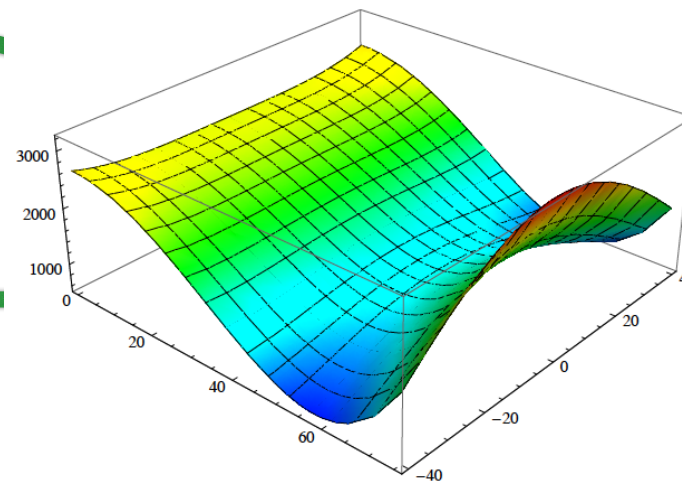
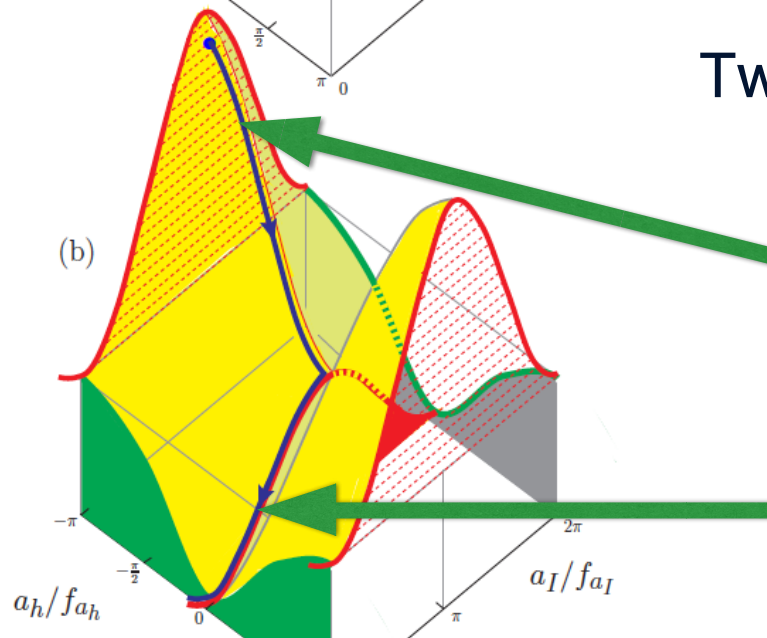






So, the radial field must roll down
the hill quickly: not inflaton
Then inflatons are angle fields:
pseudoscalars

Two angle 2-flation with 2 confining forces:
K-Nilles-Peloso (2005)



quickly.

slowly:



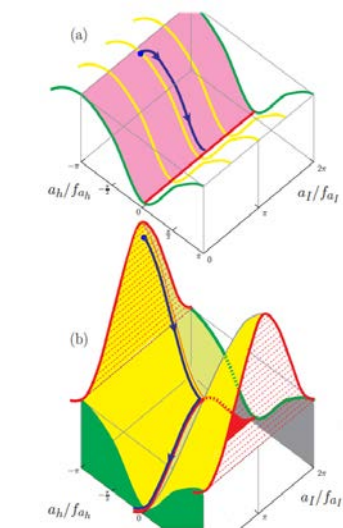
What is the number of confining groups?

Rank: $16-4=12$, $4+4+4$
So, N-flation not likely.

What is the 2-flation condition?

$$V = \Lambda_1^4 \left(1 - \cos \left[\alpha \frac{a_1}{f_1} + \beta \frac{a_2}{f_2} \right] \right) + \Lambda_2^4 \left(1 - \cos \left[\gamma \frac{a_1}{f_1} + \delta \frac{a_2}{f_2} \right] \right).$$

$$M^2 = \begin{pmatrix} \frac{1}{f_1^2} (\alpha^2 \Lambda_1^4 + \gamma^2 \Lambda_2^4), & \frac{1}{f_1 f_2} (\alpha \beta \Lambda_1^4 + \gamma \delta \Lambda_2^4) \\ \frac{1}{f_1 f_2} (\alpha \beta \Lambda_1^4 + \gamma \delta \Lambda_2^4), & \frac{1}{f_2^2} (\beta^2 \Lambda_1^4 + \delta^2 \Lambda_2^4) \end{pmatrix}.$$



$$M^2 = \begin{pmatrix} \frac{1}{f_1^2} (\alpha^2 \Lambda_1^4 + \gamma^2 \Lambda_2^4), & \frac{1}{f_1 f_2} (\alpha \beta \Lambda_1^4 + \gamma \delta \Lambda_2^4) \\ \frac{1}{f_1 f_2} (\alpha \beta \Lambda_1^4 + \gamma \delta \Lambda_2^4), & \frac{1}{f_2^2} (\beta^2 \Lambda_1^4 + \delta^2 \Lambda_2^4) \end{pmatrix}.$$

$$m_h^2 = \frac{1}{2}(A + B),$$

$$m_I^2 = \frac{1}{2}(A - B),$$

for $\alpha\delta = \beta\gamma + \Delta$

$$A = \left(\frac{\alpha^2 \Lambda_1^4 + \gamma^2 \Lambda_2^4}{f_1^2} + \frac{\beta^2 \Lambda_1^4 + \delta^2 \Lambda_2^4}{f_2^2} \right),$$

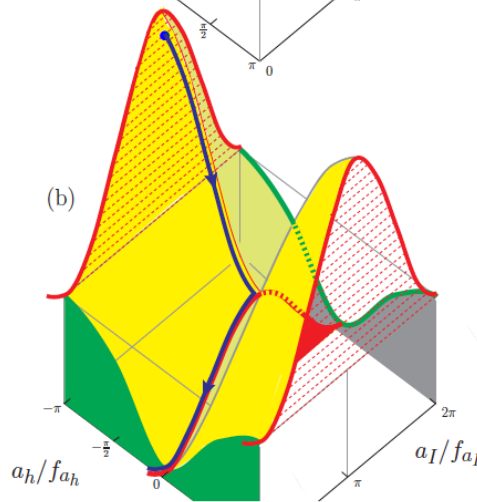
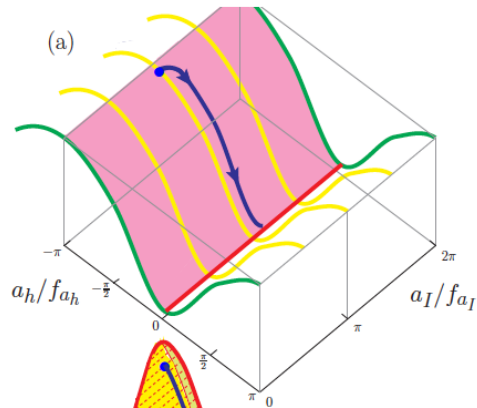
$$B = \sqrt{\left(\frac{\alpha^2 \Lambda_1^4 + \gamma^2 \Lambda_2^4}{f_1^2} + \frac{\beta^2 \Lambda_1^4 + \delta^2 \Lambda_2^4}{f_2^2} \right)^2 - 4(\alpha\delta - \beta\gamma)^2 \frac{\Lambda_1^4 \Lambda_2^4}{f_1^2 f_2^2}}.$$

$$m_{a_h}^2 \simeq (\alpha^2 + \beta^2 + \gamma^2 + \delta^2) \frac{\Lambda^4}{f^2},$$

$$m_{a_I}^2 \simeq \frac{\Lambda^4}{(\alpha^2 + \beta^2 + \gamma^2 + \delta^2) f^2 / \Delta^2}.$$

Very large decay
constant possible.





PQ quantum numbers

Delta=1

alpha=50

Prob.=1/2500?

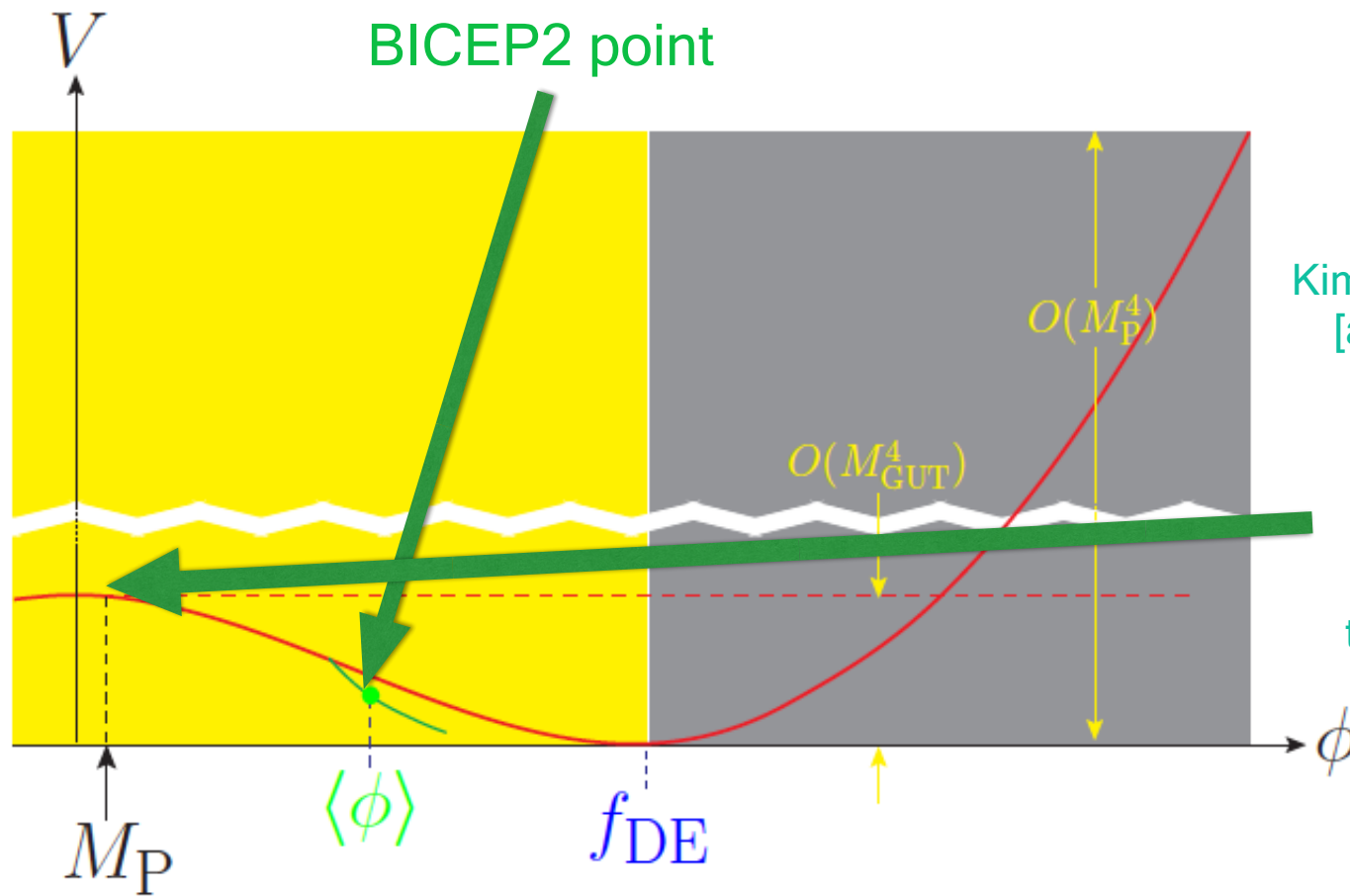
[Choi-Kim-Kyae, front. phys. 2 (2015) 83
[1410.1762]]

$$f_{a_I} = \frac{\sqrt{\alpha^2 + \beta^2 + \gamma^2 + \delta^2 f}}{\Delta}.$$



- Let us go back to the hilltop inflation.





Kim, PLB737 (2014) 1
[arXiv:1404.4022]

Kim-Mo, computer
search, starting
from the hilltop
to BICEP2, and
to the inflation end.



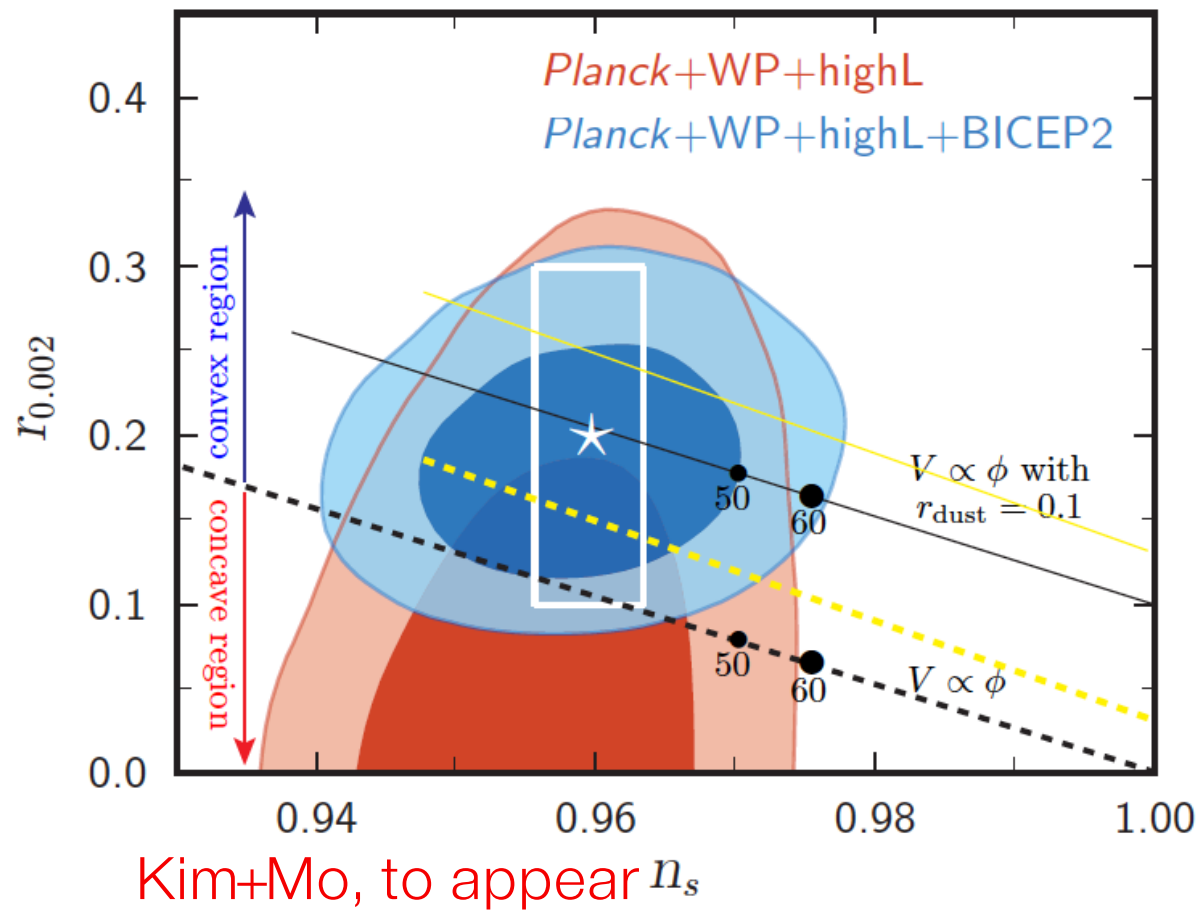
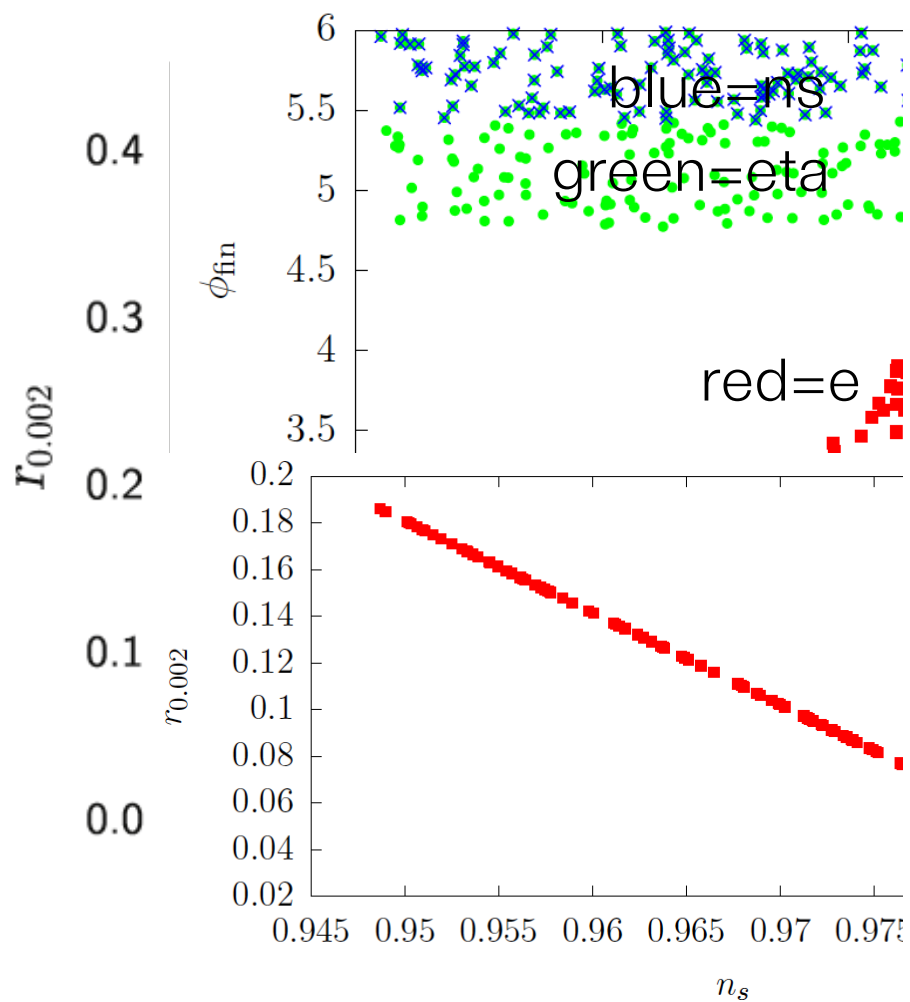


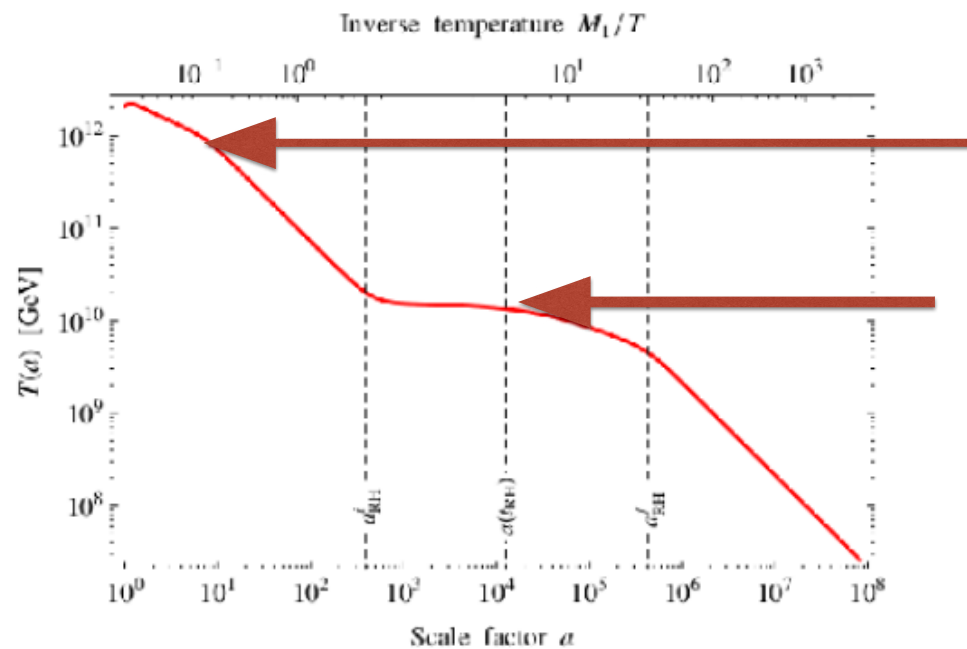
FIG. 4: The yellow curves for the e -fold numbers between 50–60 with parameters of Eq. (1) from data points of Fig. 3. The



6. PQ symmetry breaking below H_1



With the GUT scale energy density for inflation, the reheat temperature after inflation is most likely, no more less than 1/100 times the hubble parameter.



W. Buchmueller et al., 1309.7788

$$H_I \simeq 10^{14} \text{ GeV}$$

$$T_{\text{rh}} \simeq 10^{12} \text{ GeV}$$



Probably, this is the most significant
implication of BICEP2 result on
axion physics, with DW number 1:

Vissineli-Gondolo, 1403.4594.

Marsh et al, 1403.4216. 71 micro-eV???

But, it needs a calculation of axions from the system of
string-domain walls.

Florida group: 1

Cambridge group: $O(100-1000)$

Tokyo group: 25



This number is used



Small string-DW are not problematic.



The horizon scale string-DW
system is problematic.

Barr-Kim, to appear in PRL.

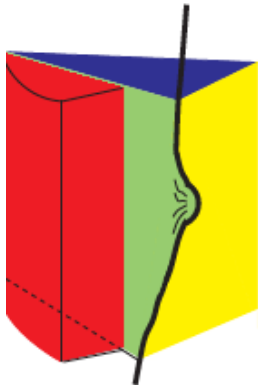




(a)



(b)



(c)



(d)

Sikivie (1982)



(a)



(b)



(c)

Vilenkin-Everett (1982);
Barr-Choi-Kim (1987)



Top-down approach, using string compactification

1. Anomalous $U(1)$ gauge symmetry.
2. Anomalous $U(1)$ becomes global $U(1)$ below the GUT scale.
3. The global $U(1)$ is broken at the axion window.
4. DW number here. By giving a VEV to $NPQ=1$ field, we obtain $NDW=1$.
5. Choi-Kim mechanism.

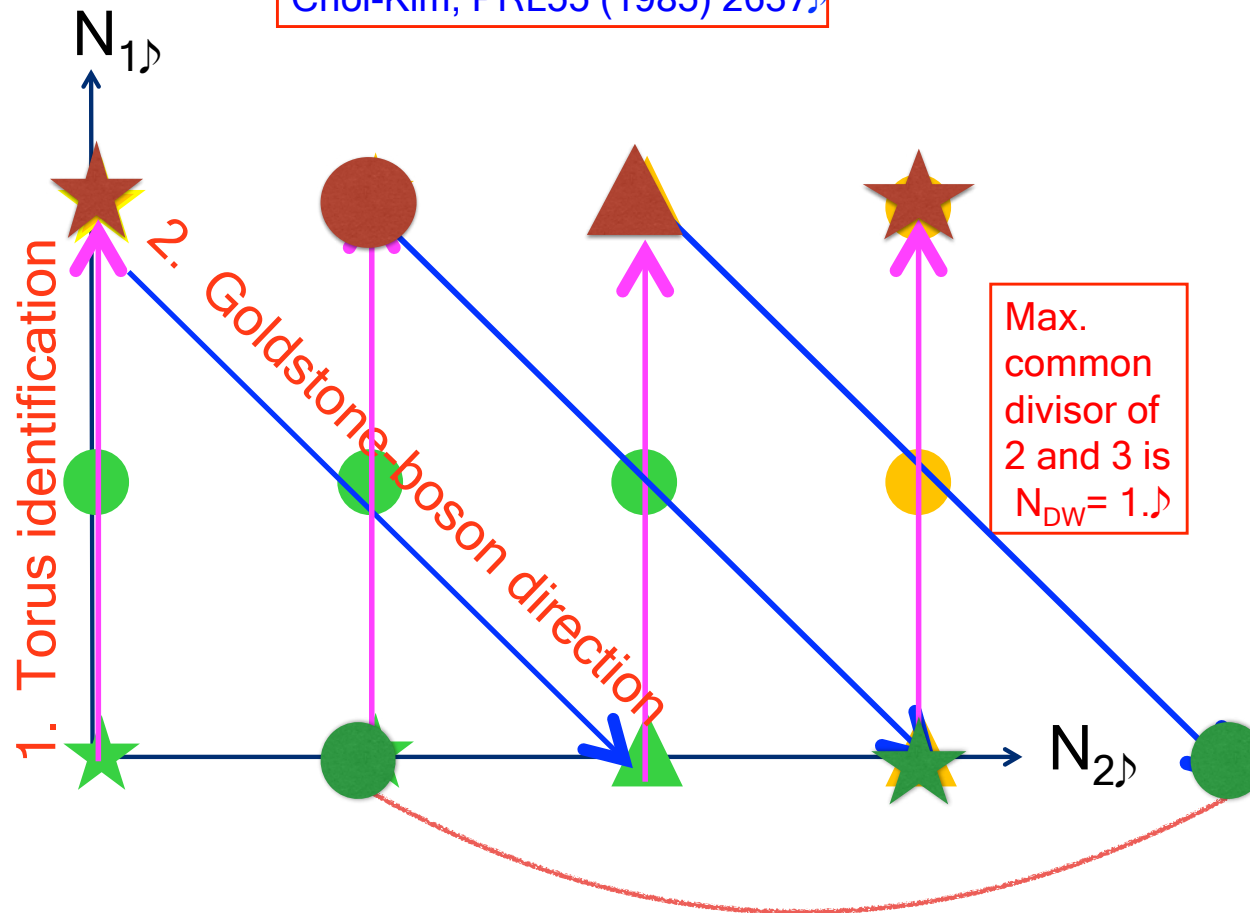


For this we need a
Goldstone boson direction



Choi-Kim mechanism

Choi-Kim, PRL55 (1985) 2637



1. Torus identification



8. Conclusion

1. ~~With~~ ~~CDM~~ ~~discrete~~ ~~any~~ ~~Dis~~ Discrete symmetry is the key.
2. ~~With~~ ~~CDM~~ ~~includes~~ ~~QCD~~ ~~axion~~. Includes QCD axion.
3. The strong CP problem is solved and the very light axion provides some CDM component.

Applied to high scale inflation

4. High scale inflation needs $NDW=1$
5. Possible in string comp. with anomalous $U(1)$
6. CAPP may get an answer to axion in the micro- to milli-eV range. in string comp. even with 10% of CDM

